SNAP Centre Workshop

Unit Conversion

Introduction

Units are fixed magnitudes of physical quantities that are used to standardize measurements. By defining a unit of measurement, any measurement made thereafter can be expressed as a multiple of that unit. When we express a quantity in terms of a standardized unit, it gives that quantity universal meaning. If a quantity is given and not accompanied by a corresponding unit, it is difficult – if not impossible – to determine what the quantity implies.

Unit Conversion

Sometimes it is necessary to convert the measurements we're working with to express them in terms of different units. Measurements can be converted within a system of units (**ex.** converting feet to inches), or from one system of units to another (**ex.** converting feet to metres).

In many cases, **conversion factors** can be used to convert measurements. They are produced by manipulating known unit equivalencies, and are used to cancel our original unit by division, and introduce our new, desired unit by multiplication.

Note: When converting units is important to keep in mind that, even though the numerical value of your measurement may change, the physical quantity being expressed by the measurement remains the same.

Example 1 1 litre is equal to 1000 millilitres.

Express this relationship as a conversion factor.

Start by stating the given information as an equation.

$$1\ L = 1000\ mL$$

Divide both sides of the equation by $1L$.

$$\frac{1\ L}{1\ L} = \frac{1000\ mL}{1\ L}$$

The equation simplifies, giving us a conversion factor.

$$\mathbf{1 = \frac{1000\ \boldsymbol{mL}}{1\ \boldsymbol{L}}}$$

Since $\frac{1000\ mL}{1\ L}$ is equal to a unit-less 1, we can safely multiply a given measurement by this conversion factor without affecting the quantity itself. This is true for all conversion factors.

Also, note that if we had divided both sides of our original equation by $1000\ mL$ instead of $1\ L$, the following conversion factor would have been our result:

$$\mathbf{\frac{1\ \boldsymbol{L}}{1000\ \boldsymbol{mL}} = 1}$$

*This illustrates that **the reciprocal of a conversion factor is also a valid conversion** factor to use, and the conversion factor used is dependent on the problem being solved.*

The language surrounding conversion factors is very important. It is not always made clear that one quantity is equal to another, as was the case in **Example 1**. For instance, saying "There are a thousand millilitres *per* liter" or "For every litre, there are one thousand millilitres" are both conversational ways of conveying the conversion factors $\frac{1000\ mL}{1\ L}$ and $\frac{1\ L}{1000\ mL}$.

Example 2 Express the mass $44\ 050\ 000$ milligrams in kilograms.

There are 1000 grams per kilogram, and 1000 milligrams per gram.

In this example, we are given a very large number of milligrams, and asked to express it in terms of kilograms, however, we are not given a conversion factor to perform the conversion directly. The conversion factors we are given verbally are $\frac{1000g}{1kg}$ *and* $\frac{1000mg}{1g}$.

Our first step will be to multiply our given quantity by the conversion factor involving milligrams. Since our goal in using a conversion factor is to cancel our original unit and replace it with another, we will multiply our quantity by the conversion factor's reciprocal, $\frac{1\ g}{1000\ mg}$.

$$(44\ 050\ 000\ \cancel{mg})\left(\frac{1\ g}{1000\ \cancel{mg}}\right)$$

$$= 44\ 050\ g$$

The result of our first step is a measurement in grams. From here, we want to use our conversion factor involving grams to find a measurement in kilograms. Again, our goal is to cancel our original unit, so we will use the reciprocal of our conversion factor.

$$= (44\ 050\ \cancel{g})\left(\frac{1\ kg}{1000\ \cancel{g}}\right)$$

$$\boldsymbol{= 44.05\ kg}$$

Using the reciprocals of the two conversion factors given, we were able to convert our measurement in milligrams into kilograms.

Notice that the end result is the same even if both conversion factors are used in a single step.

$$(44\ 050\ 000\ \cancel{mg})\left(\frac{1\ \cancel{g}}{1000\ \cancel{mg}}\right)\left(\frac{1\ kg}{1000\ \cancel{g}}\right)$$

$$\boldsymbol{= 44.05\ kg}$$

Depending on personal preference, conversions can be performed one step at a time or all at once.

Example 3 Express the length 144 inches in centimeters.

For every meter there are 100 centimeters, there are 12 inches in a foot, and 1 foot is equal to 0.3048 meters.

Looking at the information given, we aren't told how many inches there are in a millimeter or vice versa, so there is no way to convert the measurement directly.

The conversion factors we are given verbally are $\left(\frac{100cm}{1m}\right)$, $\left(\frac{12in}{1ft}\right)$, and $\left(\frac{1ft}{0.3048m}\right)$*, and – as always – we can use their reciprocals.*

Also – since inches and millimeters are units from different systems of measurement – we need to make use of a conversion factor that relates one system to the other.

Observing our conversion factors, we see that it is possible to convert inches to feet, feet to metres (a conversion that relates our unit systems), and metres to centimetres. With our individual conversions planned out in advance, we can solve the problem in a single step.

$$(144\ \cancel{in})\left(\frac{1\ \cancel{ft}}{12\ \cancel{in}}\right)\left(\frac{0.3048\ \cancel{m}}{1\ \cancel{ft}}\right)\left(\frac{100\ cm}{1\ \cancel{m}}\right)$$

After all cancellations are performed, only centimetres remain, which is exactly what we want. The only things left to do are carry out the calculation and round due to significant figures.

$$= 365.76\ cm$$

$$= \mathbf{366\ cm}$$

Note: Significant figures in this case are determined by our original measurement, since it is safe to assume that the conversion facts being used are precise to more than 3 significant figures.

Converting Measurements with Units Raised to a Power

It is often the case that a measurement will be expressed in terms of units raised to a power. Surface area and volume are both examples of such measurements, as their units can consist of length units squared and cubed, respectively.

To convert units raised to a power, the conversion factor we use needs to be raised to that same power to ensure proper unit cancellation.

Note: It is common to make the mistake of remembering to raise the units in a conversion factor to the proper power, but not the numerical values associated with them. Make sure when performing calculations that all terms in the conversion factor are raised to the proper power.

Example 4 Express the measurement $75.0\ cm^2$ in in^2.

Use the conversion factors provided in the previous question.

First, we want to plan out the steps of our conversion. In this case, our conversion is going to be the reverse of the conversion performed in ***Example 3****, with the exception that our units are now squared;* cm^2 *will be converted to* m^2, m^2 *will be converted to* ft^2*, and finally* ft^2 *will be converted to* in^2*.*

$$(75.0\ cm^2)\left(\frac{1m}{100cm}\right)^2\left(\frac{1ft}{0.3048m}\right)^2\left(\frac{12in}{1ft}\right)^2$$

$$= (75.0\ \cancel{cm^2})\left(\frac{1\ \cancel{m^2}}{10\ 000\ \cancel{cm^2}}\right)\left(\frac{1\ \cancel{ft^2}}{0.092903\ \cancel{m^2}}\right)\left(\frac{144\ in^2}{1\ \cancel{ft^2}}\right)$$

$$= 11.625\ in^2$$

$$= \mathbf{11.6\ in^2}$$

If we had only squared the units rather than the conversion factors themselves, our final answer would have been $29.5\ in^2$*, which is almost three times the area of the correct measurement!*

Sometimes measurements that are expressed in units raised to a power can be converted to measurements in units that are not raised to a power. Volume, for instance, can be described equally well by a length unit cubed (e.g. m^3 or cm^3), or a volume unit (e.g. L or mL).

Note: Litres are **derived units**, whereas metres are **base units** in their system of measurement. Derived units are defined in terms of base units, and base units are defined with respect to a constant physical quantity – in the case of metres, the distance travelled at the speed of light over a very short time interval.

Example 5 $\quad Express\ the\ volume\ 3\ m^3\ in\ mL.$

$1\ cm^3\ is\ equal\ to\ 1\ mL.$

We know that m^3 *can be converted to* cm^3 *by raising the conversion factor* $\left(\frac{100\ cm}{1\ m}\right)$ *to the third power, and we are given the conversion factor* $\left(\frac{1mL}{1cm^3}\right)$ *in the problem, so our plan for converting* m^3 *to* mL *only consists of using two conversion factors.*

$$(3\ m^3)\left(\frac{100\ cm}{1\ m}\right)^3\left(\frac{1\ mL}{1\ cm^3}\right)$$

$$= (3\ \cancel{m^3})\left(\frac{10\ 000\ \cancel{cm^3}}{1\ \cancel{m^3}}\right)\left(\frac{1\ mL}{1\ \cancel{cm^3}}\right)$$

$$= 30\ 000\ mL$$

The important thing to note in this example is that the cm^3 *to* mL *conversion factor did not need to be raised to a power to ensure proper unit cancelation. Millilitres are a unit that describe volume inherently, whereas cubic centimetres are a unit that describe volume as a length unit cubed.*

Certain units are converted using other operations, such as addition, or a combination of multiplication and addition.

Example 6 Convert $23.00℃$ to K.

$$℃ + 273.15 = K$$

Instead of multiplying, converting from degrees Celsius to Kelvin is a matter of addition or subtraction, depending on which unit you wish to express your measurement in. If we have a measurement in degrees Celsius, we simply add 273.15 *to get convert to Kelvin.*

$$\mathbf{(23.00 + 273.15)}\boldsymbol{K} = \mathbf{293.15}\boldsymbol{K}$$

Example 7 Convert $325.15\ K$ to $°\mathrm{F}$.

$$°\mathrm{F} = ℃\left(\frac{9}{4}\right) + 32$$

Converting between degrees Fahrenheit and degrees Celsius involves both multiplication and addition. The temperature we're given in the example is in Kelvin, so we must make sure to perform the conversion from Kelvin to degrees Celsius first.

$$(325.15 - 273.15)℃ = 52℃$$

Once we have our temperature in degrees Celsius, converting to degrees Fahrenheit is simply a matter of plugging our value into the equation given in the problem statement.

$$\left(52\left(\frac{9}{4}\right) + 32\right)\ °\mathrm{F} = \mathbf{149}°\mathrm{F}$$